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Hybrid Vehicle Timetable Scheduling Under Fuzzy Decision Making

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Abstract

The challenge of designing timetables for public transit is to adjust departure time to the varying passenger flow, provide high quality service, and optimize allocation of recourses. The timetable design method proposed in this study, is based on both fuzzy constraint and fuzzy goal, vehicle usage degree and passenger satisfaction, with big and small size vehicles. Passenger flow is extracted from real operation data of Shijiazhuang bus line 1, which includes history data of Intelligent Card (IC), to record passenger alighting time, and Global Position System (GPS), to record real-time location of vehicles. Decision-making on size of the vehicles to be used, and time interval at certain time state is discussed. Two strategies are investigated including minimizing total cost per minute, and random selection of vehicle size followed by maximizing the degree of fuzzy constraints. Cost of timetable evaluates from four criteria: cost of waiting time, travel time, bus services time, and vehicle drive distance. Heuristic algorithms are designed to solve this problem and integer programming is used in producing timetable. Different strategies are compared on the basis of current experimental results, and show that random selection of vehicle size followed by maximizing the degree of fuzzy constraints is a flexible and effective way in generating a timetable.

Keywords: Hybrid vehicle scheduling, Fuzzy decision making, Timetable optimization.

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1. INTRODUCTION

A timetable for public transit is a sequence of times when vehicles depart form bus station. Urban development and expansion rely on high service levels of public transit. Optimizing timetables is a rapid way to improve the service quality and rearrange limited resources. Even headway dispatch is the simplest and most common way to build a timetable. However, this often cause overcrowding at peak hours and low usage of vehicle at off peak hours. Accurate passenger flow is the key information in designing an efficient timetable. However, obtaining the specific information of passenger flow is a hard problem. Vehicle capacity is also a very important constraint, which limits the maximum number of on board passengers. The object of designing a new timetable is to provide a comfortable bus ride for passengers while allocating resources efficiently.

Previous research on obtaining the passenger flow focuses mainly on three aspects [1]: modeling passenger behaviors, using Automatic Collection Systems, and and extracting from IC card. A cellular automata-based alighting and boarding micro-simulation model for passengers in Beijing Metro stations is build according to the observation of alighting and boarding [2]; the model helps metro company in organizing passenger and evaluating function of facilities. Origin-destination of passengers are estimated from origin-only Automatic Fare Collection system [3], and many other potentials of Automatic Data Collection are discussed. Destination location for each individual can be estimated by analyzing the data from Smart Card Automated Fare Collection system [4]. The smart card is used to investigate passenger trips (user type, on-board time, travel modes) to develop a new fare system of South Korea. Instead of using one maximum fee, the new system charges based on travel distance to reduce the cost of passengers [5]. Multi-modal journeys (bus-to-underground, underground-to-bus, and bus to bus) are analyzed with Oyster smart fare payment data in London [6], to study how passenger travel in their daily life.

In this study, passenger flow is extracted, by combining data of GPS and IC, from real system. The data comes from Intelligent transport system (ITS), which is widely developed in many cities of China, and the operation data used in many researches. The reliability of the bus service is evaluated based on GPS and IC data, in which the travel time and headways at each bus stop are obtained from GPS. The number of boarding passengers is obtained from IC data [7]. Passenger flow at bus stop, and bus line can be obtained by integrating GPS and IC data [8]. Matching the boarding bus stop of passengers is much easier than an alighting bus stop, as boarding bus stop can be obtained from smart card tapping time (from IC card) and vehicle stop time (from GPS records). Deriving the passenger density and estimating passenger destination based on tapping time and boarding bus stops [9], was proposed in 2014. This is the point of departure of this study in estimating the destination of passengers.

From previous research, it can be see that detail information of passenger is fundamental of timetable scheduling. There are two common ways to built a timetable [10], frequency-based operation and timetable-based operation. Frequency-based operation is widely used in regular scheduling where the demand is stable, while timetable-based needs to adjust departure time to fit the variation of the passenger flow. Frequency determination using counted passenger data is studied in [11], in which service time is divided into small time periods (like one hour) and each period has a different frequency to adjust the variation of passenger flow; four methods are proposed to derive frequency, two are based on the maximum on-board passenger load of all bus stops, the other two are based on the ride distance of passengers; the four methods all provide schedule that reach certain load standard and avoid overcrowding. Frequency decides the crowed on-board situation. The joint optimization of price and frequency are discussed in [12]. Uncertain demand of passenger flow is taken into account in bus frequencies optimization, and described as a probability distribution function [13].

As discrete time intervals have limit values under bounded constraints [14], determination of preferred headways mainly use heuristic algorithms and are formulated as a mixed integer programming problem [15]. Decision making on time interval between two successive bus services with discrete time is easy to model and apply in real system [16]. Continuous service time is rescaled to discrete values of equal time intervals (unit of time can be 30 s, 1 min, 2 min) [17], and each discrete value is a time state that have its own value of features like: passenger flow of the line, arriving rate and leaving rate of bus stops. Hierarchical classification algorithms have been used to classify bus service time into few periods [18], time states in one period has same features [19]; but if the time span of each period is quite long, it fails to describe the traffic situation precisely. The shorter the time period, the more precisely can the travel condition be described.

Most of the timetable scheduling optimization problems in previous studies are formulated into an objective function with crisp constraints. However in the real world system, there are many reasons that affect transit system: population, weather, season, expand of city etc. Incomplete knowledge, fluctuation of passenger flow, and varying riding time challenge the performance of these approaches in deterministic environment [20]. Various kind of uncertainties can be categorized as stochastic uncertainty and fuzziness [21]. Stochastic uncertainty relates to the uncertainty of occurrences of events which lie in well defined information [22]. A fuzzy multi-objective optimization problem is formulated to model single bus line timetabling [23]. The relationship between the objective functions and decision variables is described by fuzzy reasoning schemes [24]. Weighted constraint aggregation in fuzzy optimization is specified by the preference of the decision-maker [25, 26]. A summary of understanding of fuzzy optimization and clarification of fuzzy goals and constraints are given in [22].

Our previous work [27, 28], design and optimization of timetable to meet dynamic temporary passenger flow under a fuzzy environment, followed the decision-making under fuzzy environment proposed by Bellman and Zadeh [29]. Two fuzzy constraints: the usage degree of service buses (μ_u) and the satisfaction degree of passengers (μ_s) were proposed and compared with the model under crisp constraints. The results show that the model with fuzzy constraints shorten the waiting time for passengers, and could adjust timetable better to fit the variation of passenger flow.

The model with two fuzzy constrains takes into consideration of both passengers satisfaction and bus capacity usage degree. However, time interval at peak hours and off peak hours changes a lot as passenger flow fluctuates. The variation of time interval cause dissatisfaction (especially winter in the morning) as passengers don't know how long they must wait. Boundary of headways may cause low usage degree in off peak hours (when capacity of vehicle is large), or over crowded at peak hour(when capacity of vehicle is small). Multi-vehicle in timetable scheduling have been investigated by many researchers [14, 30, 31], Minimum cost per minute is used to make decision of the size of vehicle in timetable scheduling [15]. Based on the achievements of previous research, hybrid vehicle timetable scheduling under fuzzy constraints is investigated in this study. Two vehicle sizes, B_{big} and B_{small}, are considered to reduce the variance of time interval between two successive vehicles. Two strategies, minimize total cost per minutes of objective function, random selection of vehicle size followed by maximizing the degree of fuzzy constraints is used to decide the vehicle size being used at certain time state.

Heuristic algorithms are designed to build decision space, which contain size of vehicle and value of time interval at each time state. Integer programming is used to produce the timetable based on service time and decision space. Different strategies in our model are compared on the basis of current experimental results, and show that random selection of vehicle size followed by maximizing the degree of fuzzy constraints is a better way to generate a flexible and effective timetable.

The notation and terminology are described in section 2. Models and formulations are introduced in section 3. The algorithms to calculate decision space and design timetable are given in section 4. Comparison between different models is shown in section 5. Conclusions and future research are summarized in section 6.

2. NOTATION AND TERMINOLOGY

To model the problem of timetable design, a number of quantities and operational constraints [23], are defined as follows:

2.1 Assumptions

(a) *Time State Space*: To apply the approach described in [29], the continuous service time space $\mathcal{T}$ is translated into a discrete finite $Time\ State\ Space$: $\mathcal{T}$, with $\mathcal{P} = |\mathcal{T}| < \infty$, with equivalent interval of one minute (that is, for $t_i,\ t_{i+1} \in T,\ t_{i+1} - t_i = 1$).

(b) *Time Stage Space*: $\mathcal{T}$ is divided into finite *Time Stage Space*, $\mathcal{K}$, with $\mathcal{N} = |\mathcal{K}| < \infty$, with equivalent interval $\tau \in [5, 30]$ (minutes) depending on operating requirements. The time state t corresponds to the time stage k, where k is obtained by Eq. (1).

$$k = \lceil t/\tau \rceil \tag{1}$$

(c) The *timetable* $\mathcal{S} = \{s_1, s_2, \ldots, s_{\mathcal{M}}\}$, corresponds to service time span $[s_1, s_{\mathcal{M}}]$ where s_m denotes the departing time for mth bus service from the first bus stop. The departure time of the first/last bus services $s_1/s_{\mathcal{M}}$ are predefined. $\mathcal{M}$ is the total number of services.

(d) The *decision variable*, $\delta \in \Delta$, with $|\Delta| = Q < \infty$, i.e. δ takes values in a finite set of time intervals between two successive bus stops; where $\delta_1 < \delta_2 <, \ldots, < \delta_Q$, with $\delta_{i+1} - \delta_i = 1, i = 1, 2, \ldots. Q - 1$.

(e) Bus stops are numbered from 1 to J. For $j \in \mathcal{J} = \{1, \ldots, J\}$ passengers arriving in the time interval $\delta \in \Delta$ are distributed uniformly.

(f) The maximum bus capacity $\mathcal{B}$ of big/small vehicle is $\mathcal{B}_{big}/\mathcal{B}_{small}$ [1].

(g) Dis is the distance from the first bus stop to last bus stop.

[1] In China, $\mathcal{B}$ is defined very precisely as the number of passenger seats plus the bus effective standing area (sq.m.) times 8 (this assumes that up to 8 people can stand on a square meter surface).

2.2 Constraints

To capture the traffic conditions at time sate $t \in \mathcal{T}$ corresponding to time stage $k \in \mathcal{N}$, with time interval $\delta \in \Delta$, and vehicle size $\mathcal{B}$, $\mathcal{B} \in \{\mathcal{B}_{big}, \mathcal{B}_{small}\}$. The following constraints are given.

(a) b_j^k denotes the arriving rate of bus stop j, at time stage k. The number of passengers waiting/boarding at bus stop j is $b_j^k \times \delta$, and waiting time of these passengers is $(b_i^k \times \delta^2)/2$.

(b) a_{ij}^k denotes the leaving rate of passengers boarding from bus stop i and alighting at bus stop j at time stage k. The number of passengers boarding from bus stop i and alighting at bus stop j is $b_i^k \times \delta \times a_{ij}^k$.

(c) $\mathcal{N}_j^{t,\delta}$ denotes the number of passengers still on-board the bus when it leaves bus stop j, and shown in Eq. (2).

$$\mathcal{N}_j^{t,\delta} = \mathcal{N}_{j-1}^{t,\delta} + b_j^k \times \delta - \sum_{i=1}^{j-1} b_i^k \times \delta \times a_{ij}^k \tag{2}$$

(d) t_j^k is the travel time between stops j and $j+1$ for bus depart bus stop j at time stage k.

(e) w_j^k is the weight of bus stop j at k.

(f) When vehicle capacity is $\mathcal{B}$, $\mu_{s,\mathcal{B}}(\mathcal{N}_j^{t,\delta})$ is the membership function of fuzzy goal (passenger satisfaction), evaluated at the number of passengers on-board, $\mathcal{N}_j^{t,\delta}$; $\mu_{u,\mathcal{B}}(\mathcal{N}_j^{t,\delta}, n)$ is the membership functions of constraint (vehicle usage degree), evaluated at the number of passengers on-board, $\mathcal{N}_j^{t,\delta}$ and threshold n. The shape of fuzzy constraint is adjustable with different value of n to fit the needs of scheduler and produce certain number of bus services.

(g) $\mu_{s,\mathcal{B}}^t(\delta)$, $\mu_{u,\mathcal{B}}^t(\delta)$ are the membership functions for passenger satisfaction and vehicle usage degree respectively, evaluated at $\mathcal{N}_j^{t,\delta}$, aggregated over all but the last bus stop.

(h) $\mu_{D,\mathcal{B}}^t(\delta)$ is the fuzzy set of decision, which results from the intersection of $\mu_{s,\mathcal{B}}^t(\delta)$ and $\mu_{u,\mathcal{B}}^t(\delta)$.

(i) δ_t means the optimized time interval at time state t.

(j) $C_{\mathcal{B}}^t(\delta)$ denotes cost per minute under different time interval δ vehicle at time state t with size $\mathcal{B}$.

(k) A $1 \times \mathcal{P}$ Vector $\text{Flag}_{\mathcal{B}}(t)$ is used to record the size of vehicle that should be used at time state t. A $1 \times \mathcal{P}$ Vector $\text{Flag}_{\mathcal{S}}(t)$ is used to record weather there is a bus service at time state t after timetable scheduled.

3. MODEL FORMULATION

Model 1 uses a minimum cost per minute [15], to make decisions on the type of vehicles and time interval simultaneously. Model 2 chooses the time interval and size of vehicle that has maximum fuzzy degree.

3.1 Model 1

Decision of vehicle size $\mathcal{B}$ and time interval δ at time state t is as follows:

$$\delta_t = \arg\min\{C^t_{\mathcal{B}}(\delta)\} \tag{3}$$

where

$$C^t_{\mathcal{B}_{big}}(\delta) = \frac{\sum_{j\in\mathcal{J}\backslash\{J\}}\left(\frac{1}{2}c_1 \times b^k_j \times \delta^2 + c_2 \times \mathcal{N}^{t,\delta}_j \times t^k_j + c_3 \times t^k_j + c_5 \times Dis\right)}{\delta} \tag{4a}$$

$$C^t_{\mathcal{B}_{small}}(\delta) = \frac{\sum_{j\in\mathcal{J}\backslash\{J\}}\left(\frac{1}{2}c_1 \times b^k_j \times \delta^2 + c_2 \times \mathcal{N}^{t,\delta}_j \times t^k_j + c_4 \times t^k_j + c_6 \times Dis\right)}{\delta} \tag{4b}$$

subject to:

$$k = \lceil t/\tau \rceil, \tau \in [5, 30] \tag{5a}$$

$$\mathcal{N}^{t,\delta}_j \le \mathcal{B},\ j = 1, \dots, \mathcal{J} - 1 \tag{5b}$$

$$\delta \in \Delta = \{\delta_1, \delta_2, \dots, \delta_Q\} \tag{5c}$$

$$\mathcal{B} \in \{\mathcal{B}_{big}, \mathcal{B}_{small}\} \tag{5d}$$

$$\text{Flag}_{\mathcal{B}}(t) = \begin{cases} 0 & \text{if} \quad C^t_{\mathcal{B}_{small}}(\delta) <= C^t_{\mathcal{B}_{big}}(\delta) \ \text{ at } t \\ 1 & \text{if} \quad C^t_{\mathcal{B}_{big}}(\delta) < C^t_{\mathcal{B}_{small}}(\delta) \ \text{ at } t \end{cases} \tag{5e}$$

Cost per minute for big and small vehicle are calculated respectively at each time state t, as shown in Eq. (4a) and Eq. (4b). The optimal time interval δ_t at time state t is decided by the minimum $C^t_{\mathcal{B}}(\delta)$, as shown in Eq. (3). The flag of vehicle size which brings the minimum $C^t_{\mathcal{B}}(\delta)$ is recorded in vector $\text{Flag}_{\mathcal{B}}(t)$, as shown in Eq. (5e).

3.2 Model 2

Vehicle size in Model 2 is decided randomly, then time interval is chosen by maximizing decision value of fuzzy sets, which associate with on-board passenger, vehicle size and time interval between two successive buses, shown as follows:

$$\begin{aligned} &\text{Flag}_{\mathcal{B}}(t) = round(rand) \\ &\text{Flag}_{\mathcal{B}}(t) = \begin{cases} \mathcal{B} = \mathcal{B}_{small} & \text{if} \quad \text{Flag}_{\mathcal{B}}(t) == 0 \\ \mathcal{B} = \mathcal{B}_{big} & \text{if} \quad \text{Flag}_{\mathcal{B}}(t) == 1 \end{cases} \\ &\delta_t = \arg\max_{\delta}\{\mu^t_{\mathcal{D},\mathcal{B}}(\delta)\} \end{aligned} \tag{6}$$

where

$$\mu^t_{\mathcal{D},\mathcal{B}}(\delta) = \mu^t_{u,\mathcal{B}}(\delta) \wedge \mu^t_{s,\mathcal{B}}(\delta) \tag{7a}$$

$$\mu^t_{s,\mathcal{B}}(\delta) = \sum_{j=1}^{\mathcal{J}-1} w^k_j \times \mu_{s,\mathcal{B}}(\mathcal{N}^{t,\delta}_j) \tag{7b}$$

$$\mu^t_{u,\mathcal{B}}(\delta) = \sum_{j=1}^{\mathcal{J}-1} w^k_j \times \mu_{u,\mathcal{B}}(\mathcal{N}^{t,\delta}_j, n) \tag{7c}$$

subject to:

$$k = \lceil t/\tau \rceil, \tau \in [5, 30] \tag{8a}$$

$$\mathcal{N}^{t,\delta}_j \leq \mathcal{B},\ j = 1, \ldots, \mathcal{J} - 1 \tag{8b}$$

$$\delta \in \{\delta_1, \delta_2, \ldots, \delta_Q\} \tag{8c}$$

$$\mathcal{B} \in \{\mathcal{B}_{small}, \mathcal{B}_{big}\} \tag{8d}$$

$$\sum_{j=1}^{\mathcal{J}} w^k_j = 1 \tag{8e}$$

$$w^k_j = \frac{b^k_j + \sum_{i=1}^{j-1} b^k_i \times a^k_{ij}}{\sum_{j=1}^{\mathcal{J}} (b^k_j + \sum_{i=1}^{j-1} b^k_i \times a^k_{ij})} \tag{8f}$$

In models 1 and 2, the number of onboard passengers at time state t, bus stop j with time interval δ, $\mathcal{N}^{t,\delta}_j$, which is calculated in Eq. (2), can not exceed the maximum capacity of the vehicle $\mathcal{B}$, as shown in Eq. (5b) for Model 1 and Eq. (8b) for Model 2.

In Model 2, vehicle size at time state t is decided randomly by $round(rand)$, and if $\text{Flag}_{\mathcal{B}}(t)$ equal to 0, $\mathcal{B} = \mathcal{B}_{small}$; otherwise $\mathcal{B} = \mathcal{B}_{big}$; after this the optimal time interval δ at time state t, with vehicle size $\mathcal{B}$ is decided by maximizing the decision value, as shown in Eq. in (6). Where $\mu^t_{s,\mathcal{B}}(\delta)$ and $\mu^t_{u,\mathcal{B}}(\delta)$ reflect the satisfaction degree of passengers and usage degree of the bus capacity at time state t with time interval, δ, as shown in Eq. (7b) and Eq. (7c) . $\mu^t_{x,\mathcal{B}}(\delta), x \in \{s, u\}$ is obtained by the aggregation of $\mu_{x,\mathcal{B}}(\mathcal{N}^{t,\delta}_j), x \in \{s, u\}$ at stop j, $j = 1, 2, \ldots, \mathcal{J} - 1$, weighted by w^k_j at time stage k. w^k_j is calculated by boarding and alighting number of passengers, as shown in Eq. (8f) with constraint Eq. (8e). $\mu^t_{\mathcal{D},\mathcal{B}}(\delta)$ is the fuzzy set intersection of the satisfaction and capacity usage degree, as shown in Eq. (7a).

3.3 Evaluation of Timetable

In Model 1, timetable $\mathcal{S}$ is evaluated by total cost, $Cost_{\mathcal{S}}$, shown in Eq. (9). For Model 1, consider cost of passengers and cost of bus company (with big and small size vehicle) in total. Where c_1 is the waiting time cost per hour, $\mathcal{T}_w$ is the passenger waiting time shown in Eq. (10a); c_2 is the travel time cost per hour, $\mathcal{T}_t$ is the passenger travel time shown in Eq. (10b); c_3 and c_4 are the operation costs for big and small vehicle per hour respectively, $\mathcal{T}_b$ and $\mathcal{T}_s$, stand for total service time of big and small vehicle respectively, shown in Eq. (10c) and Eq. (10d); c_5 and c_6 are the distance costs for big and small vehicle per kilometer, $\mathcal{D}_b$ and $\mathcal{D}_s$, stand for total drive distance of big and small vehicle respectively, calculate by Eq. (10e) and Eq. (10f).

$$Cost_{\mathcal{S}} = c_1\mathcal{T}_w + c_2\mathcal{T}_t + c_3\mathcal{T}_b + c_4\mathcal{T}_s + c_5\mathcal{D}_b + c_6\mathcal{D}_s. \tag{9}$$

Where:

$$\mathcal{T}_w = \sum_{t\in\mathcal{T}}\left(Flag_{\mathcal{S}}(t) \times \sum_{j\in\mathcal{J}\setminus\{J\}} \frac{1}{2} \times b_j^k \times (\delta_t)^2\right) \tag{10a}$$

$$\mathcal{T}_t = \sum_{t\in\mathcal{T}}\left(Flag_{\mathcal{S}}(t) \times \sum_{j\in\mathcal{J}\setminus\{J\}} \mathcal{N}_j^{t,\delta} \times t_j^k\right) \tag{10b}$$

$$\mathcal{T}_b = \sum_{t\in\mathcal{T}}\left(Flag_{\mathcal{S}}(t) \times Flag_{\mathcal{B}}(t) \times \sum_{j\in\mathcal{J}\setminus\{J\}} t_j^k\right) \tag{10c}$$

$$\mathcal{T}_s = \sum_{t\in\mathcal{T}}\left(Flag_{\mathcal{S}}(t) \times (1 - Flag_{\mathcal{B}}(t)) \times \sum_{j\in\mathcal{J}\setminus\{J\}} t_j^k\right) \tag{10d}$$

$$\mathcal{D}_b = \sum_{t\in\mathcal{T}} (Flag_{\mathcal{S}}(t) \times Flag_{\mathcal{B}}(t)) \times Dis \tag{10e}$$

$$\mathcal{D}_s = \sum_{t\in\mathcal{T}} (Flag_{\mathcal{S}}(t) \times (1 - Flag_{\mathcal{B}}(t))) \times Dis \tag{10f}$$

$$\mathrm{Flag}_{\mathcal{S}}(t) = \begin{cases} 1 \text{ there is a bus service } t \\ 0 \text{ there is no bus service } t \end{cases} \tag{10g}$$

$$\mathrm{Flag}_{\mathcal{B}}(t) = \begin{cases} 1 \quad \text{if} \quad \mathcal{B} = \mathcal{B}_{big} \text{ at } t \\ 0 \quad \text{if} \quad \mathcal{B} = \mathcal{B}_{small} \text{ at } t \end{cases} \tag{10h}$$

In Model 2, the average vehicle loads $\overline{\mathcal{N}}$, shown in Eq. (11a), average vehicle usage degree $\overline{\mu}_s$, shown in Eq. (11b), and average passenger satisfaction degree $\overline{\mu}_s$, shown in Eq. (11c), are used to evaluate timetable $\mathcal{S}$.

$$\overline{\mathcal{N}} = \sum_{t=s_2}^{s_{\mathcal{M}}}\left(\sum_{j\in\mathcal{J}\setminus\{J\}} \mathcal{N}_j^{t,\delta_t}\right) / ((\mathcal{J} - 1) \times (\mathcal{M} - 1)) \tag{11a}$$

$$\overline{\mu}_s = \sum_{t=s_2}^{s_{\mathcal{M}}}\left(\mu_{s,\mathcal{B}}^t(\delta)\right) / (\mathcal{M} - 1) \tag{11b}$$

$$\overline{\mu}_u = \sum_{t=s_2}^{s_{\mathcal{M}}}\left(\mu_{u,\mathcal{B}}^t(\delta)\right) / (\mathcal{M} - 1) \tag{11c}$$

3.4 Timetable Design and Performance Evaluation

The timetable of service time $\mathcal{S}$ is generated based on the decision space $\mathcal{D}$. Suppose two successive bus services depart at t and $t + \delta_t$ respectively, where $\delta_t \in \Delta$ is time interval calculated based on t. If the mth bus service departs at time state t, i.e. $s_m = t$, then the next bus service s_{m+1} will depart at time state $t + \delta_t$. The time interval δ_{s_m} between s_m and s_{m+1} is obtained from $\mathcal{D}(s_m)$. The performance of timetable is evaluated with respect to: (1) service time of vehicles $\mathcal{T}_s$, (2) waiting time of passengers $\mathcal{T}_w$, (3) travel time of passengers $\mathcal{T}_t$, (4) mean on-board passengers $\overline{\mathcal{N}}$, (5) mean satisfaction degree of passengers $\overline{\mu}_s$, (6) mean capacity usage degree of vehicles $\overline{\mu}_u$.

4. CASE STUDY

4.1 Parameters

In this section, timetables are designed separately in models 1 and 2 under a group of cases, the service time covers from 6:00 am to 10:00 pm, and each period last 2 hours and has 120 time states; the adjacent two cases have no time overlap, and the whole system has 960 time states. From cases 1 to 8 the time span are: 6:00 ∼ 8:00, 8:00 ∼ 10:00, . . . , 20:00 ∼ 22:00. The number of boarding passengers vary at different bus stops, FIGURE 1, bus stops 1 and 2 have a higher amount passenger flow in the morning, bus stops $10 - 12$ have more passenger boarding in the afternoon. very a few passenger boarding after bus stop 20.

Table 1: Value of parameters in case study

Parameters	value
service time	6:00 am to 10:00 pm
$\mathcal{T}$	{1,2, …, 960}
$\mathcal{K}$	{1,2, …, 96 }
τ	10 (minutes)
$\{\delta_1, \delta_2, \ldots, \delta_Q\}$	{2, 3, . . . , 15}(minutes)
$\mathcal{B}_{small}$ / $\mathcal{B}_{big}$	60 / 90
$\mathcal{J}$	24
Dis	13 (kilometers)
$b_j^k, a_{ij}^k, t_j^k, w_j^k$	obtained by average value at k
$[c_1, c_2, c_3, c_4, c_5, c_6]$	[8.6, 8.6, 20, 20, 15, 10]

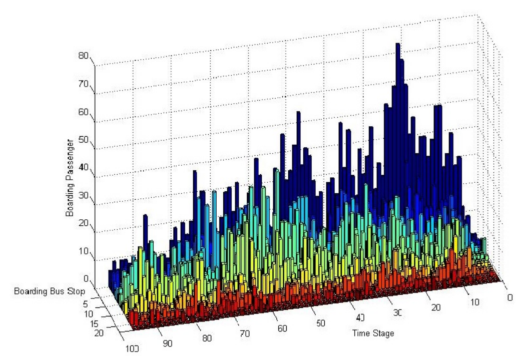


Figure 1: Passenger Flow.

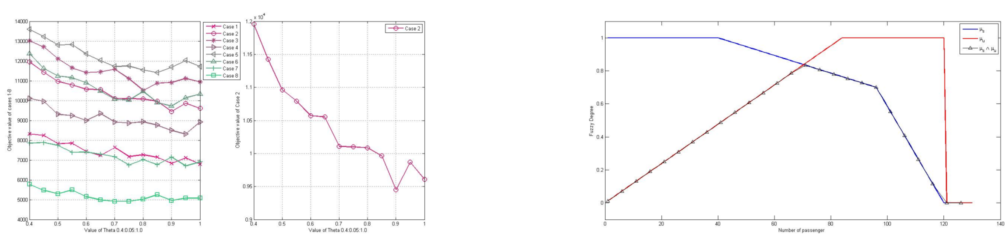
The parameters: length of time stage $\tau = 10$ (minutes), which divides 960 time states into 96 time stages. Time interval $\delta \in 2, 3, \ldots, 15$. Suppose that the passenger flow extracted from history data is not located in ShiJiaZhuang but in ShangHai, the coefficients in paper [15], can be used in this study to compare timetable produced from models 1 and 2. Coefficients are decided based on the bus line in city Shanghai[15], where the capacity of big size vehicle is $\mathcal{B}_{big} = 90$, and capacity of small vehicle $\mathcal{B}_{small} = 60$. Value of $[c_1, c_2, c_3, c_4, c_5, c_6] = [8.6, 8.6, 20, 20, 15, 10]$, as shown in TABLE 1.

Multiple vehicle size is used to reach the even loads and even headway, and the capacity constraint is to reach desired usage rate [14, 30]. The relationship between the value of objective function and usage rate of vehicle capacity are studied as follows: Set θ denotes the capacity usage rate, and $\theta = 0.4 : 0.05 : 1.0$ with step 0.05. The $Cost_{\mathcal{S}}$ for cases 1-8 under vary of θ are shown in left part of FIGURE **??**. It can be seen that the $Cost_{\mathcal{S}}$ reduce when the capacity usage rate increase, and the minimum cost of each case corresponds to $\theta \geq 0.7$; take Case 2 as an example, shown in right part of FIGURE **??**, the minimum cost of objective function in Case 2 is when capacity usage rate reach to $\theta = 0.9$.

The membership functions for the fuzzy goal μ_s given by Eq. (12). The fuzzy constraint, μ_u, in Model 2 are designed similar to the capacity usage degree in Model 1, as shown in Eq. (13). where $\mathcal{N}$ denotes the number of on-board passengers, and $\mathcal{B}$ denotes the maximum bus capacity.

$$\mu_{s,\mathcal{B}}(\mathcal{N}) = \begin{cases} 1 & 0 \leq \mathcal{N} \leq B/3 \\ -9\mathcal{N}/14B + 17/14 & B/3 < \mathcal{N} \leq 4B/5 \\ -7\mathcal{N}/2B + 7/2 & 4B/5 < \mathcal{N} \leq B \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

$$\mu_{u,\mathcal{B}}(\mathcal{N}, n) = \begin{cases} \mathcal{N}/n & 0 \leq \mathcal{N} \leq min(n, \mathcal{B}) \\ 1 & n < \mathcal{N} \leq \mathcal{B} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$



(a) Objective value under θ.

(b) Fuzzy degree with loads of passengers.

$\mu_{s,\mathcal{B}}(\mathcal{N})$ is the satisfaction degree of on-board passengers; when few passengers are on the bus (everyone has a seat), the satisfaction degree is equal to 1 as everyone is comfortable; when the number of passengers varies from $\mathcal{B}/3$ to $4\mathcal{B}/5$, the satisfaction degree slowly reduces to 0.7 as the recent passengers who have no seat are uncomfortable; however, the comfort degree of passengers will reduce sharply, when there are more than $4\mathcal{B}/5$ (and up to $\mathcal{B}$) passengers, in this situation not only standing passengers feel uncomfortable about crowd, also those seated have less space and have difficulty in alighting. The blue line in FIGURE **??** shows the shape of μ_s.

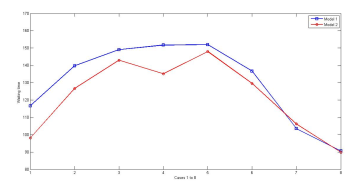
$\mu_{u,\mathcal{B}}(\mathcal{N}, n)$ is the usage degree of the bus. When the number of on-board passengers is between n and $\mathcal{B}$, the usage degree is equal to 1, When number of on-board passengers less than the threshold $min(n, \mathcal{B})$, the capacity usage degree is $\mathcal{N}/n$. n is the threshold to adjust the shape of capacity usage degree to produce certain number of bus services. In practice, the shape of fuzzy goal and fuzzy constraints capture the preference of scheduler. $\mu_{u,\mathcal{B}}(\mathcal{N}, n)$ is shown as red line in FIGURE **??** when $n = 84$.

The black line with triangle in FIGURE **??** is the intersection of $\mu_{u,\mathcal{B}}(\mathcal{N}, n)$ and μ_u, it can be seen that the fuzzy goal limits the usage degree goes up in time interval decision making.

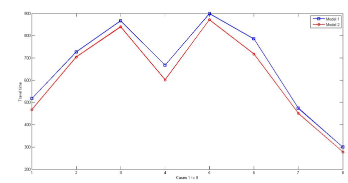
All the experiments were conducted on a laptop with an Intel core i7 2.40 GHz with 8GB RAM. The proposed Models 1 and 2 are coded in MATLAB R2012b.

4.2 Model Compare

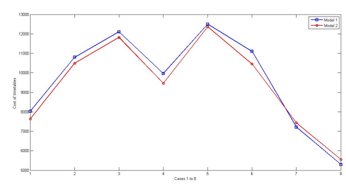
4.2.1 Hybrid vehicle



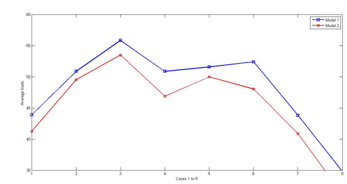
(a) $\mathcal{T}_w$



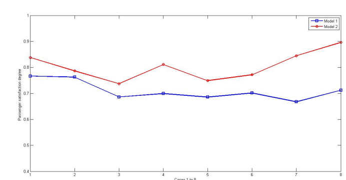
(b) $\mathcal{T}_t$



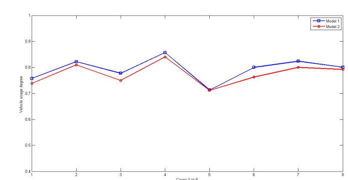
(c) $Cost_{\mathcal{S}}$



(d) $\overline{\mathcal{N}}$



(e) $\overline{\mu}_s$



(f) $\overline{\mu}_u$

Figure 2: Timetable difference of models 1 and 2 with hybrid

Figures $2(a)$ to $2(f)$ show the timetable difference in models 1 and 2 with hybrid vehicles. The total service number of model 1 and 2 are same in all cases (but vehicle size is different). FIGURE $2(a)$ shows Model 2 has much shorter waiting than Model 1, and the last two cases are closed to Model 1. FIGURE $2(b)$ shows Model 2 has much lower travel time in all cases. FIGURE $2(c)$ shows Model 2 has a smaller cost in most cases except last two case (but the costs are very close, and the last two case in Model 1 use small vehicle mainly). FIGURE $2(d)$ shows Model 2 has a more lower average loads than Model 1 in all cases, this is very important evidence that under the same number of bus services, Model 2 can be adjust to varying passenger flow better than Model 1. FIGURE $2(e)$,$2(f)$ shows that passenger in Model 2 have a higher satisfaction degree and lower capacity usage, this means that under the same traffic condition, on-board passenger in Model 2 have more space than in Model 1 .

TABLES **??** and **??** show the values of objection function $Cost_{\mathcal{S}}$, the waiting time $\mathcal{T}_w$, the travel time $\mathcal{T}_t$, the total service number $\mathcal{M}$, service number of big vehicle $\mathcal{M}_b$, service number of small

Table 2: Comparison between Model 1 and Model 2 with Case 1-8

$Case$	$\mathcal{M}$	Model 1								Model 2							
		$Cost_S$	$\mathcal{T}_w$	$\mathcal{T}_t$	$\mathcal{M}_b$	$\mathcal{M}_s$	$\overline{N}$	$\overline{\mu}_s$	$\overline{\mu}_u$	$Cost_S$	$\mathcal{T}_w$	$\mathcal{T}_t$	$\mathcal{M}_b$	$\mathcal{M}_s$	$\overline{N}$	$\overline{\mu}_s$	$\overline{\mu}_u$
1	14	8025	117	517	6	8	44	0.77	0.76	7637	98	468	3	11	41	0.84	0.74
2	16	10797	140	726	1	15	51	0.76	0.82	10494	127	704	1	15	50	0.79	0.81
3	16	12105	149	867	1	15	56	0.69	0.78	11821	143	840	1	15	53	0.74	0.75
4	15	9965	152	667	4	11	51	0.70	0.86	9454	135	601	1	14	47	0.81	0.84
5	17	12494	152	898	3	14	52	0.69	0.71	12494	152	898	3	14	52	0.69	0.71
6	15	11098	137	786	1	14	52	0.70	0.80	11098	137	786	1	14	52	0.70	0.80
7	13	7213	103	474	8	5	44	0.67	0.82	7213	103	474	8	5	44	0.67	0.82
8	12	5294	91	299	9	3	35	0.71	0.80	5294	91	299	9	3	35	0.71	0.80

vehicle $\mathcal{M}_s$, average satisfaction degree of passengers $\overline{\mu}_s$ and average usage degree of vehicles $\overline{\mu}_u$. From TABLES **??** and **??**, it can be seen that under the same environment, Models 2 has lower cost, lower waiting time and lower travel time in most of the cases. For the same service number, Model 2 has higher satisfaction degree of passengers and lower capacity usage degree of vehicles compared with Model 1. The FIGURE 2 shown the difference more clearly.

Form the case study, it can be seen that models 1 and 2 produce similar timetable when adjusting Model 2 produce the same service number with Model 1, and Model 2 works better than Model 1 in most cases. Decision making in Model 1 depending on the objective function, which choose longer time interval to have a higher vehicle usage degree that leads to lower cost. Decision making in Model 2 rely on both fuzzy goal and fuzzy constraint, first make sure that passenger has a higher satisfaction degree then choose the time interval that has a higher vehicle usage degree. The membership function of fuzzy goal and fuzzy constraint show the preference of decision maker and more flexible and adjustable.

5. CONCLUSION

Two strategies, cost minimum per minute (Model 1), random selection of vehicle size followed by maximizing the degree of fuzzy constraints (Model 2), are discussed in this for the problem of deciding the time interval and vehicle size to build a mixed vehicle size timetable. The experimental results show that adjustment of the membership functions in Model 2 yields a similar timetable and cost with the timetable proceed by Model 1. The timetable in Model 2 works better than Model 1 as it has lower loads, shorter waiting time, and higher passenger satisfaction. Model 1 is very sensitive to the objective function that it does produce small cost but unbalanced satisfaction degree of passengers and usage degree of vehicles. The decision-making in model 1 can be simulated by a fuzzy set based on the value of the objective function change with the capacity usage of the vehicle. The membership function of fuzzy goal and fuzzy constraint reflects the preference of timetable scheduler and is more robust in balancing the needs of passengers and the scheduling of vehicles.

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